Skimming: A Response to Weber and Mejía-Ramos

Matthew Inglis and Lara Alcock
Loughborough University, United Kingdom

We recently reported a study in which undergraduate students and research mathematicians were asked to read and validate purported proofs (Inglis & Alcock, 2012). In our eye-movement data, we found no evidence of the initial skimming strategy hypothesized by Weber (2008). Weber and Mejía-Ramos (2013) argued that this was due to a flawed analysis of eye-movement data and that a more fine-grained analysis led to the opposite conclusion. Here we demonstrate that this is not the case, and show that their analysis is based on an invalid assumption.

Weber and Mejía-Ramos (2013) suggested that our analysis was flawed because, after calculating what proportion of reading time a mathematician took to reach the last line of a proof (which they called an Initial Reading [IR] ratio), we took means across different tasks. Considering means, they argued, obscures reading strategy variation. Clearly, this is true in principle, and at the end of this response, we discuss what exactly is obscured in our data. First, however, we respond to Weber and Mejía-Ramos's more specific criticisms.

Neither by Subjects Nor by Items

Weber and Mejía-Ramos argued that our parametric analysis was invalid because the distribution of IR ratios is not normal; however, they took task-individual pairs as their unit of analysis. Our unit of analysis was different, as we conducted a traditional by-subjects analysis, finding that the distribution of participants' mean IR ratios *was* approximately normal. Thus our analysis, with inference to the mean time it takes mathematicians to first fixate on the last lines of these purported proofs, is valid.

Knowing this, of course, does not tell us directly about reading strategies on individual proofs, and Weber and Mejía-Ramos aimed to investigate these strategies using single IR ratios. But their method results in a statistically problematic mixture of by-subjects and by-items analyses: It allows generalization neither to the population of mathematicians nor to the population of proofs. It does not even permit conclusions about the IR-ratio distribution in the population of task-individual pairs, because the sampling then grossly violates the assumption of independence (each participant appears in six task-individual pairs; each task in 12). Thus, single IR ratios do not allow us to make desirable inferences, even though they exhibit more detail of the data.

The Absence of Fixations Versus the Presence of a Fixation

One strength of eye tracking is that it provides a lot of detail; however, a corresponding risk is that the data is noisy. The noise creates problems for Weber and Mejía-Ramos's argument that low IR ratios—early first fixations on last lines—indicate the use of a skimming strategy. Whereas we asserted that the absence of early last-line fixations implies that a skimming strategy was not used, Weber and Mejía-Ramos assumed the inverse, that the presence of early last-line fixations implies that a skimming strategy was used. This assumption is invalid. Although the absence of fixations is meaningful, the presence of a single fixation can easily be meaningless—perhaps the result of an irrelevant head movement or of returning from a blink or from an off-screen fixation. This is almost certainly the case for the very early last-line fixation in Figure 1 (from a mathematician's reading of Proof 5, as analyzed in detail by Weber and Mejía-Ramos). This fixation occurred after 0.6% of the total reading time, giving an IR ratio of 0.006. However, the point (0.006, 8) is clearly an outlier, and this participant did not skim, but rather adopted an approximately linear reading strategy for the first 60% of his or her reading time. Considering participant means reduces such misinterpretations because it decreases distortions caused by noise. Indeed, it does so in a way that is conservative: Noise can only reduce first fixation times, so "false" fixations bias the data against our conclusion.

Participant means are, of course, a crude measure of behavior. But to study that behavior directly, we can use all the eye-tracking data. Studying the time versus line-number fixation plots for Proof 5 for all twelve mathematicians[1] indicates that it is unreasonable to conclude, as Weber and Mejia-Ramos do, that half of the participants completed an initial read-through within 30% of their reading time. In fact, around 30% is the lowest proportion of reading time that any

[1] High-resolution versions of these plots are available at http://hdl.handle.net/2134/10725.

participant can be said to have taken for their initial read-through. We agree with Weber and Mejía-Ramos that to describe such behavior as skimming would "seem strange" (p. 470).

Conclusion

Weber and Mejía-Ramos reported that many mathematicians claim to skim proofs, and observed that this is surprising if the behavioral evidence suggests otherwise. We agree. We also agree that the discrepancy could indicate that we do not understand what mathematicians mean when they talk about skimming, or that this behavior appears only for more complicated proofs. We are investigating the second possibility in our ongoing work.

References

Inglis, M., & Alcock, L. (2012). Expert and novice approaches to reading mathematical proofs. *Journal for Research in Mathematics Education, 43*(4), 358–390.

Weber, K. (2008). How mathematicians determine if an argument is a valid proof. *Journal for Research in Mathematics Education, 39*(4), 431–459.

Weber, K., & Mejía-Ramos, J. P. (2013). On mathematicians' proof skimming: A reply to Inglis and Alcock. *Journal for Research in Mathematics Education, 44*(2), 464-471.

Authors

Matthew Inglis, Mathematics Education Centre, Loughborough University, Loughborough, LE11 3TU, United Kingdom; m.j.inglis@lboro.ac.uk

Lara Alcock, Mathematics Education Centre, Loughborough University, Loughborough, LE11 3TU, United Kingdom; l.j.alcock@lboro.ac.uk

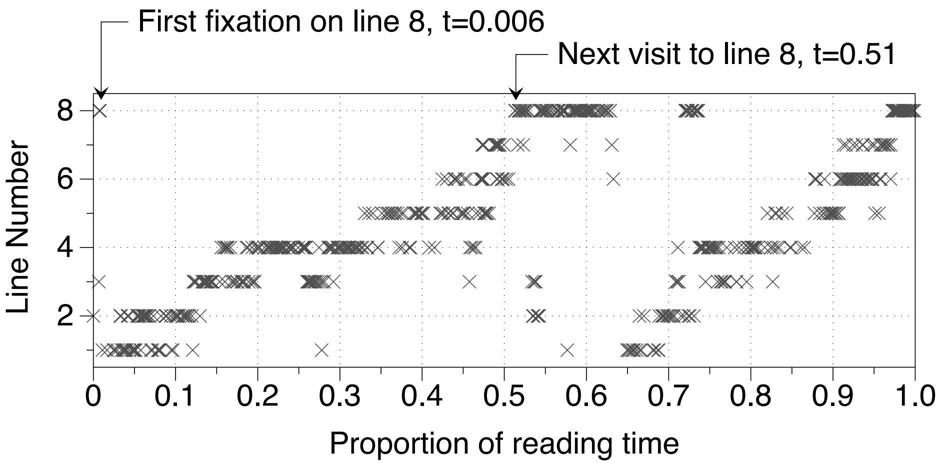

Figure 1. One mathematician's time versus line-number fixation plot for Proof 5. A point at (*x*, *y*) indicates that the participant fixated on line *y* at time *x*.